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GCE Core Mathematics C4 (6666) Paper 1

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Core Mathematics Unit C4

Introduction

This paper proved accessible to much of the candidature and afforded a typical E grade candidate enough opportunity to gain marks across the majority of questions. At the other end of the scale, there were also a few demanding marks for the A* candidates.

Examiners were impressed with the quality of work demonstrated by many of the candidates sitting this paper. The standard of algebra was very good, although a number of candidates made basic sign or manipulation errors in questions 2, 3(a), 6(c), 6(d) and 8(b). The design of the question booklet continues to help candidates to clearly present their solutions, and in almost all cases they were able to give solutions to all questions in the spaces provided.

In summary, questions 1, 2, 3, 5(a), 6(a), 6(b), 7(a), 7(b) and 8(a) were a good source of marks for the average candidate, mainly testing standard ideas and techniques; and questions 4, 5(b), 6(c), 7(e) and 7(f) were discriminating at the higher grades. Question 8(c) was the most demanding question with only about 3% of the candidature able to show that the population of meerkats could not exceed 5000.

Report on individual questions

Question 1

Most candidates attempted this question and many achieved full marks.

In part (a), most candidates were able to differentiate implicitly to gain the first three marks. A minority of candidates struggled to apply the product rule correctly on $3x^2y$. At this point a minority of candidates substituted $x=-1$ and $y=1$ in their differentiated equation, but the majority of candidates proceeded to find an expression for $\frac{dy}{dx}$ in terms of x and y, before substituting in these values. Although the majority of candidates were able to find the correct answer of $-\frac{4}{9}$, common errors in this part included sign errors either in rearranging or when substituting $x=-1$ and $y=1$ into their $\frac{dy}{dx}$ expression. A small number of candidates tried to rearrange the equation given in the question in order to make y the subject. This resulted in very few, if any, marks being awarded.

In part (b), a small minority of candidates either found the equation of the tangent and gained no marks or did not give their equation of the normal in the form $ax+by+c=0$, where a, b and c are integers, and lost the final accuracy mark.

Question 2

This question was generally well answered with around 50% of the candidature gaining all 6 marks. The majority of candidates were able to apply the integration by parts formula in the correct direction. Some candidates, however, did not assign u and $\frac{dv}{dx}$ and then write down their $\frac{du}{dx}$ and v before applying the by parts formula, which meant that if errors were made the method used was not always clear.

In part (a), $\int \sin 3x \, dx$ caused some problems for a minority of candidates who produced responses such as $\pm\cos 3x$ or $\pm 3\cos 3x$

or $\frac{1}{3}\cos 3x$. After correctly applying the by parts formula, a few candidates then incorrectly wrote down $\frac{1}{3}\int \cos 3x \, \mathrm{d}x$ as $\frac{1}{6}\cos 3x$.

Most candidates who could attempt part (a) were able to make a good start to part (b), by assigning u as x^2 and $\frac{\mathrm{d}v}{\mathrm{d}x}$ as $\cos 3x$, and then correctly apply the integration by parts formula. At this point, when faced with $\frac{2}{3}\int x\sin 3x \, \mathrm{d}x$, some candidates did not make the connection with their answer to part (a) and made little progress. Other candidates independently applied the by parts formula again, with a number of them making a sign error.

Question 3

This question was also generally well answered with about 50% of candidates obtaining all of the 9 marks available.

In part (a), a minority of candidates were unable to carry out the first step of writing $\frac{1}{(2-5x)^2}$ as $\frac{1}{4}\left(1-\frac{5x}{2}\right)^{-2}$, with the $\frac{1}{4}$ outside the brackets usually written incorrectly as either 1 or $\frac{1}{2}$. Many candidates were able to use a correct method for expanding a binomial expression of the form $(1+ax)^n$. A variety of incorrect values of *a* were seen, with the most common being either $\frac{5}{2}$, 5 or -5. Some candidates, having correctly expanded $\left(1-\frac{5x}{2}\right)^{-2}$, forgot to multiply their expansion by $\frac{1}{4}$. As expected, sign errors, bracketing errors, and simplification errors were also seen in this part.

In parts (b) and (c), most candidates realised that they needed to multiply $(2+kx)$ by their binomial expansion from part (a) and equate their x and x^2 coefficients in order to find both k and A. A small minority, however, attempted to divide $(2+kx)$ by their part (a) expansion. Other candidates omitted the brackets around $2+kx$, although they progressed as if these "invisible" brackets were really there.

In part (b), a significant minority of candidates used an incorrect method of multiplying $(2+kx)$ by the first term $\left(\text{usually } \frac{1}{4}\right)$ of their binomial expansion, and equating the result to $\frac{1}{2}$ in order to find k. In part (c), these candidates also multiplied $(2+kx)$ by the third term $\left(\text{usually } \frac{75}{16}x^2\right)$ of their binomial expansion and equated this to Ax^2 in order to find *A*.

A few candidates in parts (b) and (c) applied an alternative method of multiplying out $(2-5x)^2\left(\frac{1}{2}+\frac{7}{4}x+Ax^2+...\right)$ and equating the result to $(2+kx)$, in order to correctly find both k and A.

Question 4

At least 90% of the candidature was able to apply the volume of revolution formula correctly. Only a few candidates did not include π in their volume formula or did not square the expression for y.

The integration was well attempted and the majority of candidates recognised that the integral could be manipulated into the form $\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x$ and integrated to give their result in the form $k\ln\left(3x^2+4\right)$, usually with $k=\frac{1}{3}$. A variety of incorrect values of k were seen with the most common being either 3 or 1. A significant number of candidates integrated incorrectly to give answers such as $x^2\ln\left(3x^2+4\right)$ or $2x\ln\left(3x^2+4\right)$.

Those candidates who applied the substitution $u=3x^2+4$ proceeded to achieve $\frac{1}{3}\ln u$, and changed their x-limits of 0 and 2 to give correct u-limits of 4 and 16. Other substitutions of $u=3x^2$ or $u=x^2$, were also used, usually successfully.

Unproductive attempts were seen by a minority of candidates, such as integration by parts or simplifying $\frac{2x}{3x^2+4}$ to give $\frac{2x}{3x^2}+\frac{2x}{4}$, or integrating $2x$ and $3x^2+4$ separately and then multiplying or dividing the two results together.

The majority of candidates were able to apply the limits correctly and examiners observed the correct answer in a variety of different forms.

Question 5

This question was generally well answered with about 40% of the candidature gaining all 8 marks.

Whilst a large number of fully correct solutions were seen in part (a), there were a significant number of candidates who struggled to differentiate sine and cosine functions, with expressions such as $\frac{\mathrm{d}x}{\mathrm{d}t}=-4\cos\left(t+\frac{\pi}{6}\right)$ or $\frac{\mathrm{d}y}{\mathrm{d}t}=\frac{3}{2}\sin 2t$ being encountered frequently. Most candidates were able to apply the chain rule to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$, although the application of $\frac{\mathrm{d}y}{\mathrm{d}t}\times\frac{\mathrm{d}x}{\mathrm{d}t}$ was occasionally seen. A small proportion of candidates used the compound angle formula to rewrite x as $2\sqrt{3}\sin t+2\cos t$, or likewise the double angle formulae to rewrite y as either $3(2\cos^2 t-1)$ or $3(\cos^2 t-\sin^2 t)$ or $3(1-2\sin^2 t)$ before going on to find their $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Candidates found part (b) more challenging and a variable range of marks was awarded in this part. Although a few candidates could not proceed further from setting their $\frac{\mathrm{d}y}{\mathrm{d}x}$ to zero, most candidates appreciated that the numerator from their $\frac{\mathrm{d}y}{\mathrm{d}x}$ expression needed to be equated to zero, so resulting in the first method mark. A number of candidates who solved $\sin 2t=0$, found only one value of t (usually $t=0$) and then one point (usually $(2, 3)$). A surprisingly large number of candidates found all four correct values of t, but did not realise that they needed to use these values in order to find four sets of coordinates for (x, y). Candidates who found more than one value for t often relied on the symmetry of the diagram to find all four points rather than a full solution, and this was permitted. A surprising number of stronger candidates did not relate the diagram to the question in part (b). These candidates stopped at finding only two or three sets of coordinates when it was clear from the diagram that there was a total of four points where $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Few candidates also set the denominator equal to zero and used the resulting values of t to find erroneous coordinates.

Question 6

In part (a), virtually all candidates were able to find the *y*-value corresponding to $x = \frac{\pi}{8}$.

In part (b), most candidates were able to apply the trapezium rule correctly to find the correct estimate with the most common errors being candidates writing h as either $\frac{\pi}{10}$, $\frac{\pi}{4}$ or $\frac{\pi}{16}$; or candidates rounding incorrectly to give 1.1503. Few bracketing errors in part (b) were encountered in this session.

Part (c) provided a diverse range of solutions. Most candidates followed the advice given in the question to use the substitution of $u = 1 + \cos x$, so obtaining $\frac{\mathrm{d}u}{\mathrm{d}x} = -\sin x$ (or occasionally $\sin x$), as well as using the double angle formula for sine to process the numerator of the integral. Whilst some students found the conversion of the given integral to an expression in u beyond them, many more were able to reach an integral of the form $k\int \frac{(u-1)}{u}(\mathrm{d}u)$. Whilst most candidates reaching this stage then correctly divided through by u and integrated term by term to reach an expression of the form $k(\ln u - u)$, a few resorted to integration by parts and were generally less successful. A significant proportion of candidates lost the final accuracy mark as a result of not showing how their constant of integration could be combined with the -4 from their integration to give the stated k in the question; some found a value for k (usually 4) or some simply failed to state the final result.

In part (d), those candidates who were unable to complete part (c) often realised that they were still able to attempt part (d). The use of limits for either x or for u was generally successfully completed to obtain the value 1.227 or $4 - 4\ln 2$, but the final step of finding the error was not so successfully tackled.

Question 7

This question discriminated well across all abilities, with parts (e) and (f) being the most demanding, and those candidates who drew their own diagram being the more successful. About 15% of the candidature was able to gain all 15 marks.

Part (a) was well answered with only a few candidates adding $\overrightarrow{OB}$ to $\overrightarrow{OA}$ instead of applying $\overrightarrow{OB} - \overrightarrow{OA}$. Candidates who failed to answer part (a) correctly usually struggled to gain few if any marks for the remainder of this question.

In part (b), most candidates were able to write down a correct expression for l, but a number of candidates did not form a correct equation by writing either $\mathbf{r} = \ldots$ and so lost the final accuracy mark. (After some discussion the examiners also accepted $l = \ldots$, which is quite common, though non standard).

In part (c), most candidates were able to take the correct dot product between either $\overrightarrow{AB}$ and $\overrightarrow{AD}$ or $\overrightarrow{BA}$ and $\overrightarrow{DA}$ to obtain the correct answer of 109°. The most common error was to obtain an answer of 71° by incorrectly taking the dot product between either $\overrightarrow{AB}$ and $\overrightarrow{DA}$ or $\overrightarrow{BA}$ and $\overrightarrow{AD}$, and using this answer to obtain an answer of 109° without proper justification. A small minority of candidates applied the cosine rule correctly to achieve the correct answer. A number of candidates struggled with this part and usually took the dot product between non-relevant vectors such as $\overrightarrow{OA}$ and $\overrightarrow{OB}$ or $\overrightarrow{AB}$ and $\overrightarrow{BD}$.

In part (d), a significant number of candidates were able to obtain the correct position vector of $\overrightarrow{\mathbf{OC}} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$ by adding either $\overrightarrow{OD}$ to $\overrightarrow{AB}$ or $\overrightarrow{OB}$ to $\overrightarrow{AD}$. A few candidates also achieved the correct result by arguing that the midpoints of the two diagonals of a parallelogram are coincident. Occasionally the incorrect answer of $\overrightarrow{\mathbf{OC}} = \pm(4\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ was given, which is a result of taking the difference between $\overrightarrow{OD}$ and $\overrightarrow{AB}$.

Candidates who were successful in part (e) found the area of the parallelogram either by finding the area of triangle *ABD* using $\frac{1}{2}bd\sin A$ and doubling the result or by applying a method of base $\times$ perpendicular height. The most common error in part (e) was for candidates to find the product of lengths *AD* and *AB*.

Candidates who were successful in part (f) usually found the shortest distance by multiplying their AD by $\sin 71^\circ$ (or equivalent). Those candidates who multiplied AB by $\sin 71^\circ$ did not receive any credit. A few candidates attempted to use vectors to find $\left|\overrightarrow{\mathrm{DE}}\right|$, where E is the point where the perpendicular from D meets the line l, often spending considerable time for usually little or no reward.

Question 8

In part (a), the majority of candidates were able to write down the correct identity to find their constants correctly, although a few candidates forgot to express their answer as a partial fraction as requested in the question.

A significant minority of candidates who completed part (a) correctly, made no attempt at part (b). Most candidates, however, were able to separate the variables, although some did this incorrectly, or did not try. The majority used their part (a) answer and integrated this to obtain an expression involving two ln terms. Although many integrated their expression correctly, some made a sign error by integrating $\dfrac{1}{5-P}$ to obtain $\ln(5-P)$. Those candidates who integrated $\dfrac{1}{5P}$ and $\dfrac{1}{25-5P}$ to $\dfrac{1}{5}\ln 5P$ and $-\dfrac{1}{5}\ln(25-5P)$, respectively, tended to find subsequent manipulation more difficult. A significant number of candidates did not attempt to find a constant of integration – with some omitting it from their working whilst others referring to "$+c$" and not attempting to use the boundary condition of $t=0$ and $P=1$ to find its value. Most candidates were able to apply the subtraction (or sometimes the addition) law of logarithms correctly for their expression but a number of candidates struggled to correctly remove the logarithms from their integrated equation, with incorrect manipulation of $\ln\left(\dfrac{P}{5-P}\right) = \dfrac{1}{3}t - \ln 4$ leading to $\dfrac{P}{5-P} = e^{\frac{1}{3}t} - 4$ usually seen.

A significant number of those candidates who removed logarithms correctly were able to manipulate their result to make P the subject of their equation, although a number of these candidates could not make the final step of manipulating $P = \dfrac{5e^{\frac{1}{3}t}}{(4+e^{\frac{1}{3}t})}$ into $P = \dfrac{5}{(1+4e^{-\frac{1}{3}t})}$. Of all the 8 questions, question 8(b) was the most demanding in terms of a need for accuracy, and about 20% of candidates were able to score all 8 marks in this part.

Very few candidates gained the mark in part (c). Many were able to show that P could not be equal to 5, and some of them also looked at what happens to P as t approaches infinity, but then failed to point out that the function was strictly increasing. Few candidates were able to state $1 + 4e^{-\frac{1}{3}t} > 1$ implied $P < 5$, but some of them did not go on to give a conclusion in relation to the context of the question.

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